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Effects of dislocations on channeling radiation from a periodically bent crystal

Juby George[1], A P Pathak[1,3] and A V Solov'yov[2]

[1] School of Physics, University of Hyderabad, Central University PO, Hyderabad-46, India
[2] Frankfurt Institute of Advanced Studies, Johann Wolfgang University, Frankfurt am Main, Germany

E-mail: appsp@uohyd.ernet.in

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Abstract
In this paper the effects of dislocations on the positron channeling in a periodically bent crystal are studied. We begin with the unified treatment of the longitudinal and transverse motion of the particle. We then separate out the Schrödinger equation into longitudinal and transverse motions. The variation in effective potential and frequency in the different regions of dislocation affected channels is found. The wavefunctions of positrons channeled in the perfect and the dislocation affected channels are found and the channeling and dechanneling probabilities are calculated. The angular and spectral distributions of radiation intensity are calculated and compared with those of normal channeling. The calculations are carried out with varying values of dislocation density and varying undulator wavelength.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Ever since its discovery about 30 years back, channeling radiation has been investigated extensively both theoretically and experimentally [1–12]. The radiation was observed for the first time for positrons and later on for electrons. Emission of channeling radiation by these particles is of great importance in accelerator based research in general and atomic and condensed matter physics in particular. Initially, the observation of this radiation seemed difficult because of its low radiation frequency and the presence of other incoherent radiations like bremsstrahlung. The important breakthrough came with the realization that the relativistic effects shift the emitted photon energy range to keV or MeV from MeV or GeV channeled positrons and electrons respectively.

One of the main applications of ion channeling is in defect studies. Real crystals are never perfect and particles propagating through them can 'see' their presence through the effects these defects will produce in the solids. The study on the effects of defects on charged particle propagation has been an area of research for a long time. Some experiments have been done to explore their applications [5]. The most important example of defects that produces distortion in the channel is dislocations. This distortion, leading to curvature in the channels, alters the particle trajectory and can lead to dechanneling for large distortions [6]. The effects of these dislocations on channeling radiation have been studied both theoretically and experimentally [2] and one finds an increase in channeling radiation frequency and decrease of intensity with increase of the distortions induced by these dislocations.

Channeling in a periodically bent crystal is of recent interest in connection with the undulator problem. A crystalline undulator is basically a periodically bent channel with ultrarelativistic charged particles undergoing channeling through it. In a crystalline undulator, in addition to the channeling radiation, there occurs undulator radiation, which is due to the periodic bending of the crystallographic planes. It serves as an efficient source for coherent high energy photon emission [13–21]. The parameters of the undulator can be tuned by varying the energy and type of projectile and by choosing different crystallographic channels. Also a wide range of frequencies and bending amplitudes in crystals allows one to generate crystalline undulator radiation with energies from the eV to the MeV region. Several groups have studied this theoretically [16–21] and a few others have applied this for making undulators [22–30].

The necessary conditions to be satisfied by a crystalline undulator to become a source of radiation are discussed by

[3] Author to whom any correspondence should be addressed.

various authors [16, 31]. These are given by

$$\begin{aligned} &C = 4\pi^2\varepsilon a/U'_{\max}\lambda_u^2 < 1 \qquad \text{stable channeling}\\ &d \ll a \ll \lambda_u \qquad \text{large amplitude regime}\\ &N = L/\lambda \gg 1 \qquad \text{large number of undulator periods}\\ &L < \min[L_d(C), L_a(\omega)] \qquad \text{account for channeling}\\ &\qquad \text{and photon attenuation}\\ &\Delta\varepsilon/\varepsilon \ll 1 \qquad \text{low radiative losses} \end{aligned} \tag{1}$$

where a is the amplitude of bending of the channel, $\varepsilon = \gamma mc^2$ the energy of the particle, λ_u is the wavelength of the undulator, L the undulator length, L_d the dechanneling length, L_a the attenuation length and d the interplanar spacing.

Stable channeling of a projectile in a periodically bent crystal occurs if the maximum centrifugal force $F_{cf} = \gamma mc^2/R_{\min}$ ($R_{\min}$ being the minimum curvature of radius of the bent channel) is less than the maximal restoring force due to the interplanar field F_{int}; i.e., $C = F_{cf}/F_{int} < 1$. A crystalline undulator should be considered in the high amplitude regime. In the limit $a/d > 1$, the undulator and channeling radiation frequencies are well separated. The term 'undulator' implies the number of periods to be large so that the emitted radiation spectrum is narrow with well separated peaks.

The fourth condition in equations (1) puts a severe limitation on the allowed values of crystalline undulator length L due to dechanneling and attenuation, represented by dechanneling length L_d and attenuation length L_a respectively. A particle entering the channel undergoes scattering by electrons and nuclei of the crystal. The dechanneling effect stands for a gradual increase in the transverse energy of a channeled particle due to these inelastic collisions. At the distance L_d, from the entrance point, the particle gains a transverse energy higher than the planar potential barrier and leaves the channel. L_a is defined as the scale at which the intensity of the photon flux is decreased by a factor e due to the process of absorption and scattering. Commenting on the last condition in equations (1), the coherence of undulator radiation is only possible when the energy loss $\Delta\varepsilon$ of the particle during its passage through the undulator is small.

Various methods have been proposed to realize such a crystalline undulator. It can be done either by using ultrasonic waves [17] or by gradient crystals [22–24] or by using substrates with periodically deposited strips of alternating stresses [25]. This last method has been tried recently in the works of Guidi *et al* and Lanzoni *et al* [27–29]. Methods like making regularly spaced grooves on crystal surfaces [26, 27, 30] and using crystals with periodic surface deformations [30] have also been proposed to achieve periodic bending in a crystal. However, an actual crystalline undulator has still not been completely realized. On the other hand, all the theoretical models for crystalline undulators vis-a-vis channeling radiation have ignored the presence of defects and damage, which is invariably present in the materials. As mentioned above, the dislocations are the most important defects, having long range effects on channeling phenomena because of the distortions they produce. These distortions are also likely to have very significant effects on the analysis of

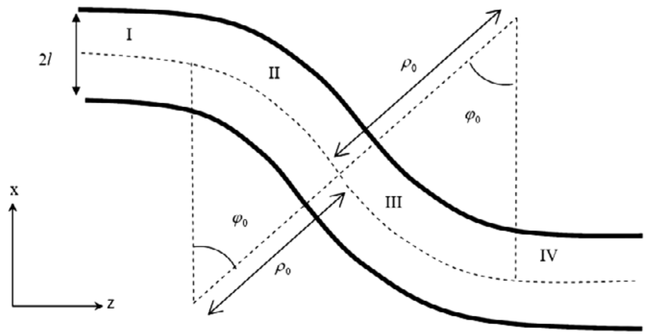

Figure 1. The model for the channel affected by dislocation. Instead of the straight channels as in the figure, for the present study the whole region is considered as periodically bent.

the channeling in periodically bent crystals. We have reported in one of our recent works [32] the effects of dislocations on channeling. The shifting of potential minima and the variation of channeling/dechanneling probabilities with the radius of curvature of the dislocation affected regions are proof of the fact that effects of defects cannot be neglected in the channeling studies. In the present analysis, we investigate the effects of dislocations on a particle propagating in a periodically bent crystal.

A periodically bent channel with a reasonable amplitude of bending a ($a \gg d$) is considered. Both the dislocation affected region and the periodically bent channel are represented by their radii of curvature and wavelengths (represented by λ_d and λ_u respectively) and we consider the modulation of these effects of dislocations over the periodicity of the channel. Two cases of high and low dislocation densities are considered. The varying dislocation densities change these modulation effects. We divide the channel into four regions: beginning and ending with perfect periodically bent regions with two dislocation affected regions between them as shown in figure 1.

2. Effects of dislocations on channeling in a periodically bent crystal

Let us consider a crystal whose planes are periodically bent following a perfect harmonic shape $x(z) = a\sin(k_u z)$. The transverse and longitudinal coordinates of a channeled particle in such a periodically bent crystal can be written as [20]

$$\tilde{x} = x - a\sin(k_u z) \tag{2}$$

where

$$k_u = \frac{2\pi}{\lambda_u}. \tag{3}$$

We consider now the effects of dislocation on such a typical channel situated at some finite distance from the dislocation core, outside the dechanneling cylinder [7]. The dislocation induced distortions in the crystallographic channels are divided into two regions [32], namely regions II and III, which smoothly join the perfect region I and region IV, as shown in figure 1. The centrifugal forces act in opposite directions in regions II and III. Here ρ_0 corresponds to the

Table 1. Parameters of a dislocation affected region for channeling in the ⟨110⟩ direction for Si.

Dislocation density (cm^{-2})	r_0 (nm)	R_d (nm)	λ_d (nm)
10^{10}	0.5×10^2	10.28×10^5	6.28×10^2
10^9	1.58×10^2	10.266×10^6	9.92×10^2
10^8	0.5×10^3	10.28×10^7	6.28×10^3

radial coordinate of the channel center as measured from the origin and φ_0 is the corresponding angular coordinate.

Consider the first region, which is part of the normal periodically bent channel. The Schrödinger equation can be written as

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\Psi^{\mathrm{I}}(x,z)+U(x)\Psi^{\mathrm{I}}(x,z) = E^{\mathrm{I}}\Psi^{\mathrm{I}}(x,z) \quad (4)$$

where E^{I} is the total energy and the transverse potential $U(x)$ is given by

$$U(x) = V_0\tilde{x}^2 = V_0(x - a\sin(k_u z))^2. \quad (5)$$

Now consider the dislocation affected regions of the channel. Centrifugal force proportional to μ^2/ρ^2 becomes operative in the curved regions of the channel. Here $\mu\hbar$ is the angular momentum with $\mu^2 = l(l+1)$ with l as the orbital angular momentum quantum number and ρ is the radius of curvature of the channel. For the two regions of dislocation affected channel these forces are acting in directions opposite to each other. Schrödinger equations for both these regions are written in terms of the polar coordinates ρ and φ.

2.1. *Low dislocation density* ($\lambda_d > \lambda_u$)

Now assume that a finite number of undulator periods are there in one length of the dislocation affected region of the channel. Let x_d be the amplitude of the wave corresponding to the dislocation affected region with a wavelength λ_d and a phase difference of ϕ between the dislocation affected channel and the undulator wave. Now we can write

$$\lambda_d = n\lambda_u \quad (6)$$

$$k_u = nk_d. \quad (7)$$

Both these waves can be written in the form

$$r_1 = a\sin(nk_d z) \quad (8)$$

$$r_2 = x_d\sin(k_d z + \phi). \quad (9)$$

Superposition of the waves gives

$$r = r_1 + r_2 = A\sin(k_d z + \Phi) \quad (10)$$

where A and Φ are the effective amplitude and phase of the final wave and are given by

$$A^2 = a^2 + x_d^2 + 2ax_d\cos[(n-1)k_d z - \phi] \quad (11)$$

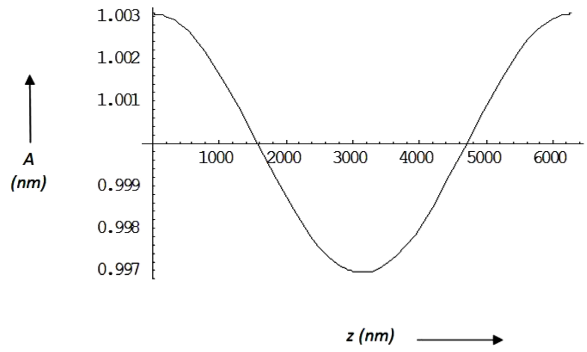

Figure 2. The change in amplitude of oscillations with respect to the depth of the crystal. The amplitude is no longer constant but varies periodically with respect to the depth.

Table 2. The relation between a periodically bent channel and the dislocation affected region for channeling in a periodically bent region at a dislocation density of 10^8 cm^{-2}.

a (nm)	λ_u ($\times 10^3$ nm)	R_u (nm)	E (MeV)	x_d (nm)
1	3.14	2.5×10^5	142.363	2.198×10^4
10	3.14	2.5×10^4	14.236	2.198×10^3
100	3.14	2.5×10^3	1.412	2.198×10^2

$$\tan\Phi = \frac{a\sin[(n-1)k_d z] + x_d\sin\phi}{a\cos[(n-1)k_d z] + x_d\cos\phi}. \quad (12)$$

Consider region II, i.e. the first curved part of the dislocation affected channel. The Schrödinger equation for this region in terms of the polar coordinates ρ and φ is given by

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi^{\mathrm{II}}(\rho,\varphi) + U(\rho)\Psi^{\mathrm{II}}(\rho,\varphi) = E^{\mathrm{II}}\Psi^{\mathrm{II}}(\rho,\varphi). \quad (13)$$

With the channel periodically bent ρ_0 can now be written as

$$\tilde{\rho_0} = \rho_0 - x_d\sin(k_d z) + A\sin(k_u z). \quad (14)$$

The variation of both the amplitude of bending and the radius of curvature of the dislocation affected region depends on both the waves: the dislocation affected region and the undulator represented by λ_d and λ_u. The following tables show how these parameters depend on each others' values. Table 1 gives the variation of the parameters of the dislocation affected region: radius and length of the curved region, $R_d = \rho_0$ and $\lambda_d = 2z$ respectively, with dislocation density. We have [6]

$$R_d = \frac{2\pi^2 r_0^2}{b\cos^3\varphi} \qquad \lambda_d = 2z = \frac{2\pi r_0}{\cos\varphi}.$$

For channeling in the Si(110) direction, the Burgers vector $b = 3.84$ Å. Table 2 gives the range of various parameters of the periodically bent channel affected by dislocations corresponding to a dislocation density of 10^8 cm^{-2}.

Figures 2 and 3 show the variation of the amplitude of bending of the channel and the radius of curvature of the dislocation affected region respectively. From figure 2, it is found that the amplitude is no longer constant but varies periodically with respect to the depth. Figure 3 shows that

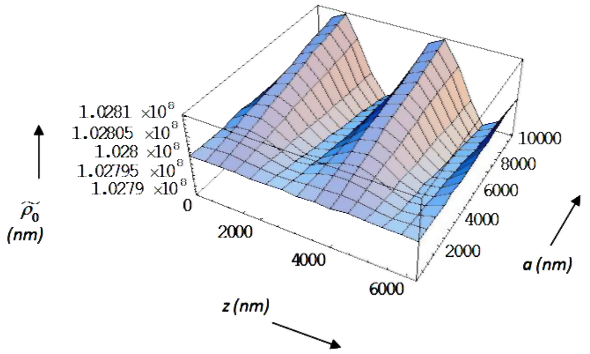

Figure 3. The change in radius of curvature of the dislocation affected channel with respect to amplitude of bending and depth. Here we find that the larger the value of a, the larger is the variation of $\tilde{\rho}_0$ with z.

the larger the value of a, the larger is the variation of $\tilde{\rho}_0$ with z. We consider the low dislocation density case of 10^8 cm^{-2}. The radius R_d of the dislocation affected region is taken as 10.28×10^7 nm and λ_d as 6.28×10^3 nm. It is interesting to note that the larger the variation of a with the length of the dislocation affected region, the smaller the effective radius of curvature. This is due to the fact that when ρ_0 follows the curvature to λ_u to become $\tilde{\rho}_0$ the effective radius of the curved region is reduced because of the larger curvature of the undulator curve.

We consider the phase difference, $\Phi = 0$ and $n = 2$ for the sake of simplicity of calculation. The potential equation can be written as

$$U(\rho) = V_0(\rho - \tilde{\rho}_0)^2. \tag{15}$$

The effective potential after including the centrifugal force term can be written as

$$V_\mathrm{eff} = V_0(\rho - \tilde{\rho}_0)^2 + \frac{\hbar^2}{2m}\frac{\mu^2}{\rho^2}. \tag{16}$$

Let

$$\xi = \rho - \tilde{\rho}_0. \tag{17}$$

Simplifying and solving, we get the effective potential as

$$V_\mathrm{eff}(\xi) = \frac{\hbar^2}{2m}\left[\frac{\lambda}{\tilde{\rho}_0{}^4}(\xi - \tilde{a_p})^2 + U_\mathrm{min}\right] \tag{18}$$

where

$$\lambda = 3\mu^2 + b^4\tilde{\rho}_0{}^4 \tag{19}$$

$$\tilde{a_p} = \frac{\tilde{\rho}_0\mu^2}{\lambda} \tag{20}$$

$$U_\mathrm{min} = \frac{\mu^2}{\lambda\tilde{\rho}_0{}^2}(\lambda - \mu^2) \tag{21}$$

$$b = \left(\frac{m\omega}{\hbar}\right)^{1/2}. \tag{22}$$

The above equations show that the shape of the potential changes due to the effects of dislocations as before [32]. The minima of the potential gets shifted due to these effects of dislocation in the channel and it is dependent on the undulator parameters as well.

The frequency of oscillation in region II is obtained as

$$\omega' = \left(\frac{\hbar}{m}\right)\sqrt{\frac{\lambda}{\tilde{\rho}_0{}^4}}. \tag{23}$$

Consider region III, i.e. the second curved part of the dislocation affected channel. The centrifugal force is acting in the opposite direction to that in the first curved region. Solving this part of the distorted region as in region II, we get the effective potential as

$$V_\mathrm{eff}(\xi) = \frac{\hbar^2}{2m}\left[\frac{\lambda'}{\tilde{\rho}_0{}^4}(\xi + \tilde{a}'_p)^2 + U'_\mathrm{min}\right] \tag{24}$$

where

$$\lambda' = -3\mu^2 + b^4\tilde{\rho}_0{}^4 \tag{25}$$

$$\tilde{a}'_p = \frac{\tilde{\rho}_0\mu^2}{\lambda'} \tag{26}$$

$$U'_\mathrm{min} = -\frac{\mu^2}{\lambda'\tilde{\rho}_0{}^2}(\lambda' + \mu^2). \tag{27}$$

The minimum of the potential is shifted in the opposite direction to that in region II and is dependent on the undulator parameters. The frequency of oscillation in region III is obtained as

$$\omega'' = \left(\frac{\hbar}{m}\right)\sqrt{\frac{\lambda'}{\tilde{\rho}_0{}^4}}. \tag{28}$$

Region IV is again dislocation free and like any other channel which is periodically bent. It has only transmitted wave and the wavefunction is similar to that in region I.

We have four regions and the wavefunctions corresponding to these regions are written as [32]

$$\Psi^\mathrm{I}(x,z) = A_0X_0^\mathrm{I}\mathrm{e}^{\mathrm{i}k_0z} + \sum_{n=0} B_nX_n^\mathrm{I}\mathrm{e}^{-\mathrm{i}k_nz} \tag{29}$$

$$\Psi^\mathrm{II}(\rho,\varphi) = \sum_{m=0} R_m^\mathrm{II}[C_m\mathrm{e}^{\mathrm{i}\mu\varphi} + D_m\mathrm{e}^{-\mathrm{i}\mu\varphi}] \tag{30}$$

$$\Psi^\mathrm{III}(\rho,\varphi) = \sum_{m=0} R_m^\mathrm{III}[G_m\mathrm{e}^{\mathrm{i}\mu\varphi} + H_m\mathrm{e}^{-\mathrm{i}\mu\varphi}] \tag{31}$$

$$\Psi^\mathrm{IV}(x,z) = X_n^\mathrm{IV}I_n\mathrm{e}^{\mathrm{i}k_nz}. \tag{32}$$

To find the reflection and transmission coefficients, we use the boundary conditions across the three boundaries and these are obtained as

$$|R|^2 = \frac{(-\mu^2 + k^2\tilde{\rho}_0{}^2)^2\sin^2(2\mu\varphi_0)}{4k^2\mu^2\tilde{\rho}_0{}^2\cos^2(2\mu\varphi_0) + (\mu^2 + k^2\tilde{\rho}_0{}^2)^2\sin^2(2\mu\varphi_0)} \tag{33}$$

$$|T|^2 = \frac{4k^2\mu^2\tilde{\rho}_0{}^2}{4k^2\mu^2\tilde{\rho}_0{}^2\cos^2(2\mu\varphi_0) + (\mu^2 + k^2\tilde{\rho}_0{}^2)^2\sin^2(2\mu\varphi_0)}. \tag{34}$$

The above equations (33) and (34) are the dechanneling and channeling coefficients respectively. Comparing with the usual dislocation affected channel we find that a dislocation in a periodically bent crystal changes the channeling and

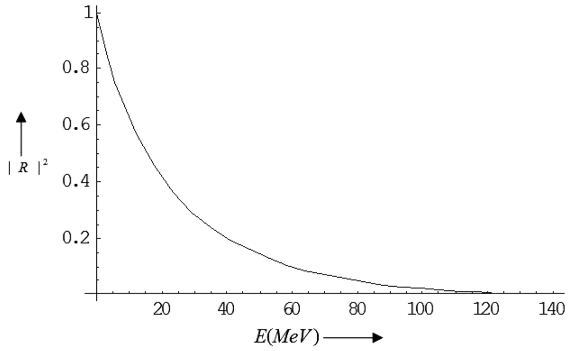

Figure 4. The change in the dechanneling probability with incident energy.

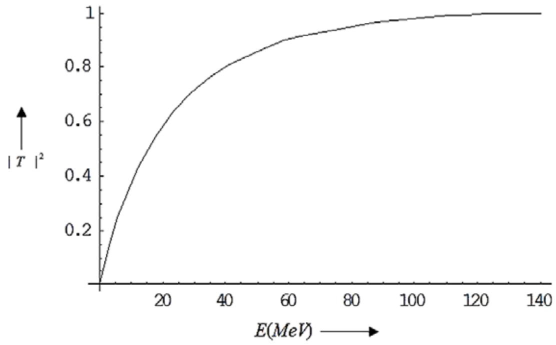

Figure 5. The change in the channeling probability with incident energy.

dechanneling coefficients by the parameters of the crystalline undulator.

For a dislocation density of 10^8 cm^{-2}, the value of $\rho_0 = 10.28 \times 10^7$ cm. The variation of the dechanneling and channeling probabilities corresponding to this fixed value of radius of curvature is the same as that of any normal channel affected by dislocations. Figures 4 and 5 show the variation of these probability values with incident energy. It is found that transmission of particles is maximum for incident energies close to 140 MeV. Consequently, we choose an incident energy of 150 MeV to demonstrate the variation of the channeling probability with the amplitude of bending and depth as seen in figure 6. This variation directly follows the variation of the modulated radius of curvature $\tilde{\rho}_0$ with these parameters.

Now we proceed to find the spectral distribution of radiation intensity. The probability of transition from an initial state i to the final state f per unit time is determined by the well known formula

$$W_{\mathrm{fi}} = \frac{4\pi^2 e^2}{\hbar V} \sum_{\vec{q}} |\vec{q}|^{-1} |\vec{\alpha}_{\mathrm{fi}} \cdot \vec{e}_k|^2 \delta(\omega_{\mathrm{fi}} - \omega) \tag{35}$$

where V is the volume of the system, and $\vec{q}$ and $\vec{e}_k$ are the wavevector and polarization vector of a quantum of electromagnetic field.

$$\hbar\omega_{\mathrm{fi}} = E_{n\mathrm{i}} - E_{n\mathrm{f}}. \tag{36}$$

The matrix elements $\vec{\alpha}_{\mathrm{fi}}$ are given by

$$\vec{\alpha}_{\mathrm{fi}} = \delta_{\sigma_{\mathrm{iz}}\sigma_{\mathrm{fz}}} \delta_{p_{\mathrm{iy}}, p_{\mathrm{fy}} + \hbar q_y} \vec{D}_{\mathrm{fi}} \tag{37}$$

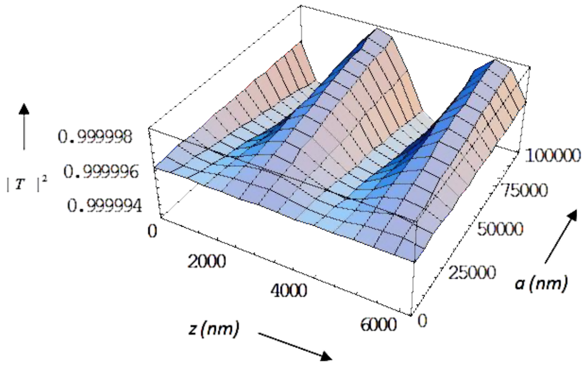

Figure 6. The change in channeling probability with z and the amplitude of bending a.

$$\vec{D}_{\mathrm{fi}} = -\mathrm{i} x_{\mathrm{fi}} (\Omega_{\mathrm{fi}}, 0, q_x \beta) \tag{38}$$

$$\Omega_{\mathrm{fi}} = \frac{\omega}{1 - \beta\cos\theta} \tag{39}$$

$$x_{\mathrm{fi}} = \int_{\infty}^{-\infty} x S_{n_\mathrm{f} E_\mathrm{f}}(x) S_{n_\mathrm{i} E_\mathrm{i}}(x)\, \mathrm{d}x \tag{40}$$

where S_{nE} are oscillatory wavefunctions which obey the Schrödinger equation given by

$$\left[-\frac{\hbar^2}{2E} \frac{\mathrm{d}^2}{\mathrm{d}x^2} + U(x) \right] S_E(x) = E S_E(x). \tag{41}$$

Let us define a vector of polarization $\vec{e}_1$ in the plane having the wavevector $\vec{q}$ and the z-axis and a vector $\vec{e}_2 \perp \vec{e}_1$ in the plane having the axes x and y. If φ and θ are the azimuthal and polar angles of the wavevector $\vec{q}$,

$$\vec{e}_1 = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta) \tag{42}$$

$$\vec{e}_2 = (-\sin\varphi, \cos\varphi, 0). \tag{43}$$

The summation in equation (35) is written in the integral form as

$$W_{\mathrm{fi}} = \frac{e^2}{2\pi\hbar} \int (|\vec{\alpha}_{\mathrm{fi}} \cdot \vec{e}_1|^2 + |\vec{\alpha}_{\mathrm{fi}} \cdot \vec{e}_2|^2) |\vec{q}|^{-1} \delta(\omega_{\mathrm{fi}} - \omega)\, \mathrm{d}\vec{q}. \tag{44}$$

Solving this we get the transition probabilities as

$$\frac{\mathrm{d}W_{\mathrm{fi}}}{\mathrm{d}\Omega} = \frac{e^2 x_{\mathrm{fi}}^2 \Omega_{\mathrm{fi}}^3}{2\pi\hbar(1-\beta\cos\theta)^4} [(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi] \tag{45}$$

$$\frac{\mathrm{d}W_{\mathrm{fi}}}{\mathrm{d}\omega} = x_{\mathrm{fi}}^2 \Omega_{\mathrm{fi}}^2 \frac{e^2}{\hbar} \left[1 - 2\frac{\omega}{\omega_m} + \left(\frac{\omega}{\omega_m} \right)^2 \right] \tag{46}$$

$$\frac{\mathrm{d}I_{\mathrm{fi}}}{\mathrm{d}\Omega} = \frac{e^2 x_{\mathrm{fi}}^2 \Omega_{\mathrm{fi}}^4}{2\pi(1-\beta\cos\theta)^5} [(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi] \tag{47}$$

$$\frac{\mathrm{d}I_{\mathrm{fi}}}{\mathrm{d}\omega} = 3 I_{\mathrm{fi}}^{(0)} \left[1 - 2\frac{\omega}{\omega_m} + 2\left(\frac{\omega}{\omega_m} \right)^2 \right] \tag{48}$$

where

$$\omega_m \approx 2\gamma^2 \Omega_{\mathrm{fi}} \tag{49}$$

$$I_{\mathrm{fi}}^{(0)} = \tfrac{4}{3} e^2 \Omega_{\mathrm{fi}}^4 \gamma^4 x_{\mathrm{fi}}^2. \tag{50}$$

Unlike in the other dislocation problems, the value of Ω_{fi} changes periodically since the shifted equilibrium axis is periodically bent. Also, the frequencies of oscillation in both the dislocation affected regions are not constant. Figure 4 shows the combined change in the spectral distribution of radiation intensity due to the periodicity of the channel and dislocation in comparison with the straight channel where $s = 4e^2\Omega_{\mathrm{fi}}^4\gamma^4 x_{\mathrm{fi}}^2$. We find a considerable amount of change mostly due to the change in the frequency of oscillation.

2.2. High dislocation density ($\lambda_{\mathrm{d}} < \lambda_{\mathrm{u}}$)

Now consider a case where the dislocation affected region is a (small) part of one undulator wavelength. Such a situation arises when the dislocation density is high. In this case, the periodicity of the crystalline undulator is affected in just a few regions of the undulator wavelength. Just like in the case of the straight channel, where a part of the channel is shifted due to the dislocation, here a region of the periodically bent channel is shifted.

Equation (2) can be written as

$$\tilde{x} = x - a\sin(k_{\mathrm{u}}vt) \tag{51}$$

which gives

$$\ddot{\tilde{x}} = \ddot{x} + ak_{\mathrm{u}}^2 v^2 \sin(k_{\mathrm{u}}vt) \tag{52}$$

keeping

$$\frac{1}{R} = ak_{\mathrm{u}}^2 \sin(k_{\mathrm{u}}vt) \tag{53}$$

we get

$$\ddot{\tilde{x}} = \ddot{x} + \frac{v^2}{R}. \tag{54}$$

Rewriting equation (52), we get

$$\gamma m\ddot{\tilde{x}} = \dot{p} + \gamma m\frac{v^2}{R} \tag{55}$$

where

$$\dot{p} = -\frac{\partial H}{\partial\tilde{x}} \tag{56}$$

where H is given by

$$H = \sqrt{c^2p^2 + m^2c^4} + qeU(\tilde{x}); \tag{57}$$

solving the above equation, we get

$$\frac{\partial H}{\partial\tilde{x}} = qe\frac{\partial U(\tilde{x})}{\partial\tilde{x}}. \tag{58}$$

Hence equation (55) can be written as

$$\ddot{\tilde{x}} + \frac{qe}{m\gamma}U(\tilde{x}) - \frac{v^2}{R}\tilde{x} = 0. \tag{59}$$

Integrating from $\tilde{x}$ to $\tilde{x}_m$ and solving, we get the maximum amplitude of oscillation of the particle as

$$\tilde{x}_m = \frac{m\gamma v^2}{qeV_0R} \tag{60}$$

Table 3. The various parameters of the undulator and dislocation affected regions at high dislocation density, when $\lambda_{\mathrm{u}} = 2\lambda_{\mathrm{d}}$.

Dislocation density (cm^{-2})	λ_{d} (nm)	λ_{u} (nm)	a (nm)	R_{u} (nm)	E (MeV)
1.5×10^9	1.66×10^3	3.32×10^3	1	2.8×10^5	150
			10	2.8×10^4	15
			100	2.8×10^3	1.5

and the new equilibrium axis is shifted to

$$\tilde{x}_0 = \frac{m\gamma v^2}{2qeV_0R}. \tag{61}$$

In this case, since $\lambda_{\mathrm{d}} < \lambda_{\mathrm{u}}$, the dislocation affected region is just a part of the undulator wavelength. Hence we can consider the region affected as almost like a straight channel. The slight periodicity of the region is reflected in the sine terms of x_m and x_0. The period of oscillation of the particle in the channel can be found in the same way as in a straight channel [6].

Solving equation (59), we get

$$\mathrm{d}t = \frac{\mathrm{d}\tilde{x}}{\left[\frac{2v^2}{R}\tilde{x} - \frac{2qe}{m\gamma}U(\tilde{x})\right]^{1/2}}. \tag{62}$$

Solving the above equation, we get the period of oscillation as

$$T = \left(\frac{m\gamma}{2qeV_0}\right)^{1/2} \sin^{-1}\left\{1 - \frac{2qeV_0R}{m\gamma v^2}\cos(k_{\mathrm{u}}z)\right\}. \tag{63}$$

Table 3 shows the values of the various parameters of the channel affected with dislocation. In this case, the dislocation density is high in comparison with the previous case for a similar range of E and its value is found to be close to $1.5 \times 10^9\ \mathrm{cm}^{-2}$.

3. Results and discussion

It is clear from the calculations that the curvature of the channel due to dislocation shifts the potential minima even for a periodically bent channel. This results in the change of the frequency of oscillations in different regions of dislocation affected channel. In the case of periodic bending the shift in potential minima and the frequency of oscillations depends on the undulator parameters which is varying with the length of the undulator. That means the shift is not a constant throughout the undulator but varies periodically with the length.

We first considered a case of low dislocation density with $\lambda_{\mathrm{d}} = 2\lambda_{\mathrm{u}}$. The frequency of channeling radiation as well as the undulator radiation are affected by the dislocations as expected. Figures 2 and 3 show the change in the amplitude of oscillation and radius of curvature of the dislocation affected regions respectively. The periodic variation of the amplitude is modulated by dislocation affected curvatures of the two regions. We chose a dislocation density in the lower regime of $10^8\ \mathrm{cm}^{-2}$ to get the corresponding radius of curvature of the dislocation affected region as 10.28×10^7 nm. The incident

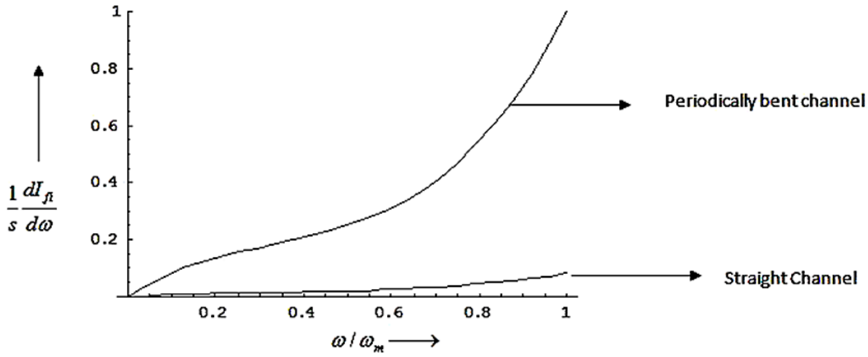

Figure 7. The change in the spectral distribution of radiation intensity due to the periodicity of the channel. $s = 4e^2\Omega_{\rm fi}^4\gamma^4x_{\rm fi}^2$.

energy is found to be around 142 MeV to get most of the particles channeled. The amplitude of bending is modulated by the curvatures of the dislocation affected regions of the channel. It is found that this amplitude of bending is no longer a constant, but varies periodically with the amplitude and wavelength of the dislocation affected region. Also, the radius of curvature of the dislocation affected region changes with the amplitude of bending and wavelength of the undulator. The larger the value of the amplitude of bending, the larger the variation of the radius of curvature with the length of the undulator.

The reflection and transmission coefficients calculated from the boundary conditions correspond to the values of dechanneling and channeling coefficients respectively. These are dependent on the undulator parameters via their dependence on $\tilde{\rho}_0$ as seen in equations (33) and (34). Figures 4–6 show the variation of these probability values with incident energy and with the depth and amplitude of bending of the channel. It is noted from figure 6 that the variation of the channeling probability follows the variation in the curved regions modulated one over the other.

The spectral distribution of radiation intensity is calculated and plotted in figure 7. We find a considerable amount of change in the spectral distribution of radiation intensity. It is to be noted that the change in radiation parameters is mainly due to the periodic bending of the channel; i.e., the crystalline undulator plays a major role in the case of low dislocation density.

For the sake of completeness and comparison, we consider the case of $\lambda_{\rm d} < \lambda_{\rm u}$ corresponding to high dislocation density. All possible ranges of values for both the undulator and dislocation wavelengths are discussed in detail and corresponding channeling parameters are calculated. The period of oscillation of the particle in the dislocation affected region for high values of dislocation density is found. It is observed from the calculations that in this case of high dislocation density the change in various parameters of channeling in the dislocation affected region is least affected by undulator parameters.

4. Conclusions

We have developed a quantum mechanical model for the effects of dislocations on positron channeling along a periodically bent channel. The shift in potential minima due to the dislocation in the channel is found for the low dislocation density case of $\lambda_{\rm d} > \lambda_{\rm u}$. The channeling and dechanneling coefficients are found from the boundary conditions across the three boundaries in the channel which separate the dislocation affected regions and are found to vary with the changes in the amplitude of bending of the channel. The spectral distribution of radiation intensity is found and compared with that for channeling in a straight channel. The results for all possible ranges of values of both the dislocation and undulator wavelengths are compared and corresponding channeling parameters are calculated. The period of oscillation of the particle in the channel is found for the high dislocation density case of $\lambda_{\rm d} < \lambda_{\rm u}$. It is seen from the entire calculations that for low dislocation density the crystalline undulator parameters play a major role, whereas in the case of high dislocation density the undulator parameters have minimal effects on the radiation. Since no crystal is perfect and study of crystalline undulators is incomplete without the consideration of defects like dislocations, the present study is crucial for investigations related to crystalline undulators.

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